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Week 3 Homework

As always, show both your code and your results. Learning to work with simulation output and display it in a useful way is important too!

In the cheese data, the retailer corresponding to the third level of RETAILER ("ATLANTA - WINN DIXIE") has a display coefficient in a regression of log(volume) on an intercept, log(price), and DISP that is negative. Most would regard this as absurd (displays of a product suppress sales). Suppose you want to impose this constraint on the model, namely that β_{display} is > 0. This can be achieved by restricting the prior support.

$$p(\beta|y, X) \propto \ell(\beta, \sigma^2)p(\beta, \sigma^2)$$

$$\beta \sim N(\bar{\beta}, A^{-1})I(\beta_{\text{display}} > 0)$$

$$\sigma^2 \sim \text{as before}$$

In the equation above, I( ) means and indicator function (1 if true 0 if not).

Show that the posterior for this model is the same as the model in the classnotes and in BSM, except that it is restricted to the region of the beta space with the display coefficient > 0. Hint: write down the prior density and ask yourself – does the normalizing constant matter?

How can you draw from this posterior?

One solution: draw from the posterior in problem 1 and discard draws that have a negative display coefficient.

More efficient solution: break the beta vector into two parts: $\beta' = (\beta_0', \beta_{\text{display}})$ where β_0 is the intercept and log price coefficient.
Implement an MCMC strategy for this model using three (rather than two) steps:

i) $\beta_{\text{display}} | \beta_0, y, X, \sigma^2$
ii) $\beta_0 | \beta_{\text{display}}, y, X, \sigma^2$
iii) $\sigma^2 | \beta, y, X$

i) is a univariate normal distribution truncated from below by 0.
ii) is a non-truncated bivariate normal distribution.
iii) Is the same as in problem 1.

Use normal theory to compute the conditional distributions.

If

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \right)$$

$$\Sigma^{-1} = V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

Then

$$\theta_1 \middle| \theta_2 \sim N\left(\mu_1 - V_{11}^{-1} V_{12} (\theta_2 - \mu_2), V_{11}^{-1}\right)$$

Notice that you can compute the relevant moments from elements of the inverse of Σ (V above)!

Implement both approaches in R and how me your posterior distributions from each approach. Note: you can use the "inefficient method" as a check on your second method as both should have the same posterior as the equilibrium distribution of the chain.