

## Week 4 Homework

If you are producing simulation output, it is imperative that you display the distribution that you are simulating from. Sequence plots and histograms are far more meaningful than sample moments!

1. Gelman and Meng (1991) (on course web site) provide an interesting example of a very non-normal distribution whose conditionals are normal!! (see section 2 of the article on p.125 for the joint density and the forms of the conditionals).
  - i). plot the bivariate density using the `image` and `contour` commands. You need to define a grid on the horizontal and vertical axes and evaluate the joint density at each point in the grid. Say `grid1` is the vector of values for the horizontal axis and `grid2` is for the vertical axis and `den` is a matrix of dimension `length(grid1) x length(grid2)` with values of the density evaluated at `grid1[i], grid2[j]`, then `image(grid1,grid2,den)` will plot an image where the colors denote height (like a topographical map). See the help file for `rmixGibbs` for examples of the use of `image` and `contour`. Try  $A = .5$ ,  $B = 0$ ,  $C_1 = C_2 = 3$ . Experiment with the grid so that it is centered where the density is highest and covers the region where the density is appreciable.
  - ii). Write code to implement a Gibbs sampler to sample from this density. Generate approximately a random sample from this density of size 1000 by making 100,000 draws from the Gibbs sampler and taking every 100<sup>th</sup> draw.
  - iii). use `rmixGibbs` to fit a mixture of normals to the data generated in part ii). Try 1 component, 2 components and 5 components. Plot the Bayes estimator of the density (posterior mean of the density evaluated at a grid – use the same grid as in part i.). To compute the Bayes estimator of the density you will have to average a mixture of normals over draws (see slide 60 of the classnotes). Use `bayesm` routine, `lndMvN`, to evaluate the log of a multivariate normal density and, therefore, by exponentiating, the density itself.
2. If a discrete Markov Chain has a symmetric transition probability matrix, what is the stationary distribution of the chain? Hint: think time reversibility!