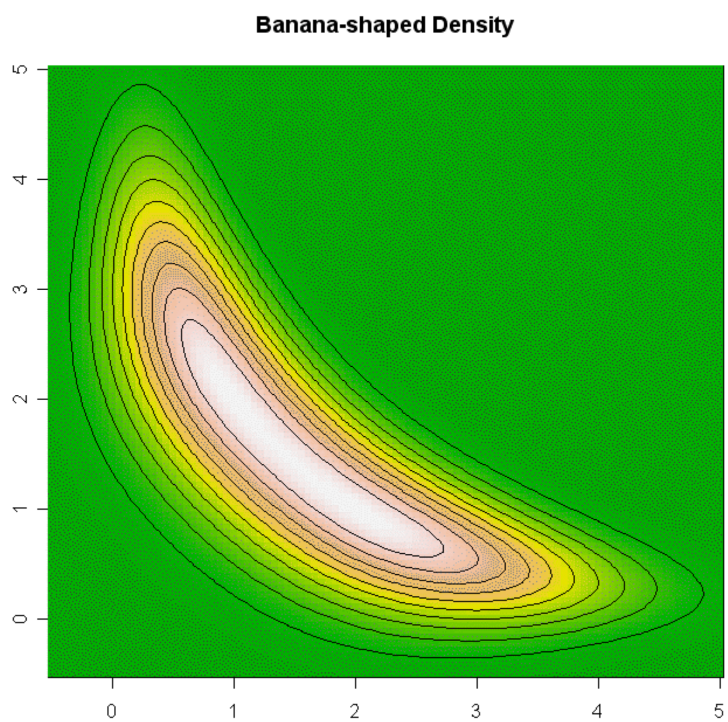
269
Week 4 Homework Solutions

If you are producing simulation output, it is imperative that you display the distribution that you are simulating from. Sequence plots and histograms are far more meaningful than sample moments!

1. Gelman and Meng (1991) (see misc articles section of course web site) provide an interesting example of a very non-normal distribution whose conditionals are normal!! (see section 2 of the article on p.125 for the joint density and the forms of the conditionals).

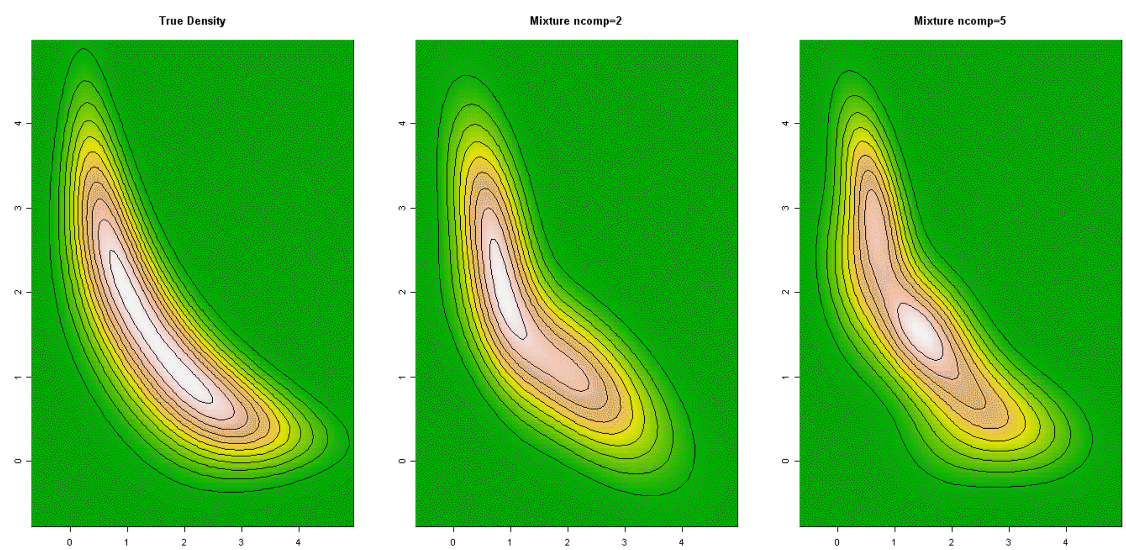
   i). plot the bivariate density using the image and contour commands. You need to define a grid on the horizontal and vertical axes and evaluate the joint density at each point in the grid. Say grid1 is the vector of values for the horizontal axis and grid2 is for the horizontal axis and den is a matrix of dimension length(grid1) x length(grid2) with values of the density evaluated at grid1[i], grid2[j], then image(grid1,grid2,den) will plot an image where the colors denote height (like a typographical map). See the help file for `rnmixGibbs` for examples of the use of image and contour. Try A= .5, B=0, C1=C2=3. Experiment with the grid so that it is centered where the density is highest and covers the region where the density is appreciable.



ii)). Write code to implement a Gibbs sampler to sample from this density. Generate approximately a random sample from this density of size 1000 by making 100,000 draws from the Gibbs sampler and taking every 100^{th} draw.

see hw _4_q1.R file for code

iii). use `rnmixGibbs` to fit a mixture of normals to the data generated in part ii). Try 1 component, 2 components and 5 components. Plot the Bayes estimator of the density (posterior mean of the density evaluated at a grid – use the same grid as in part i.). To compute the Bayes estimator of the density you will have to average a mixture of normals over draws (see slide 60 of the classnotes). Use bayesm routine, `lndMvn`, to evaluate the log of a multivariate normal density and, therefore, by exponentiating, the density itself.



see hw_4_q1.R for code

2. If a discrete Markov Chain has a symmetric transition probability matrix, what is the stationary distribution of the chain? Hint: think time reversibility!

Let P be the transition matrix and π be the stationary distribution of the chain. If π is stationary, it has to be time reversible.

i.e. $\pi_i P_{ij} = \pi_j P_{ji}$

(1)

If P is symmetric,

$P_{ij} = P_{ji} \forall i, j$

(2)

Thus, P_{ij} and P_{ji} can be cancelled out of equation (1). Then, equation (1) reduces to

$\pi_i = \pi_j$

(3)

Since equation (3) is valid for every *i* and *j*, it must be that π is a <u>discrete uniform distribution</u>.