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Homework 5 – Solutions

1. The quantity of interest to us is $\delta = E_\pi\left[h(\theta)\right]$. Unless $h(\theta)$ is known to be linear,

$$E_\pi\left[h(\theta)\right] \neq h\left(E_\pi(\theta)\right) \qquad (1)$$

.

$\hat{\delta} = \frac{1}{R}\sum_r h\left(\theta^r\right)$ is clearly an unbiased estimator of $\delta = E_\pi\left[h(\theta)\right]$. And because of the above inequality (1), in general

$\tilde{\delta} \neq h\left(\frac{1}{R}\sum_r \theta^r\right)$ unless we know that $h(\theta)$ is linear.

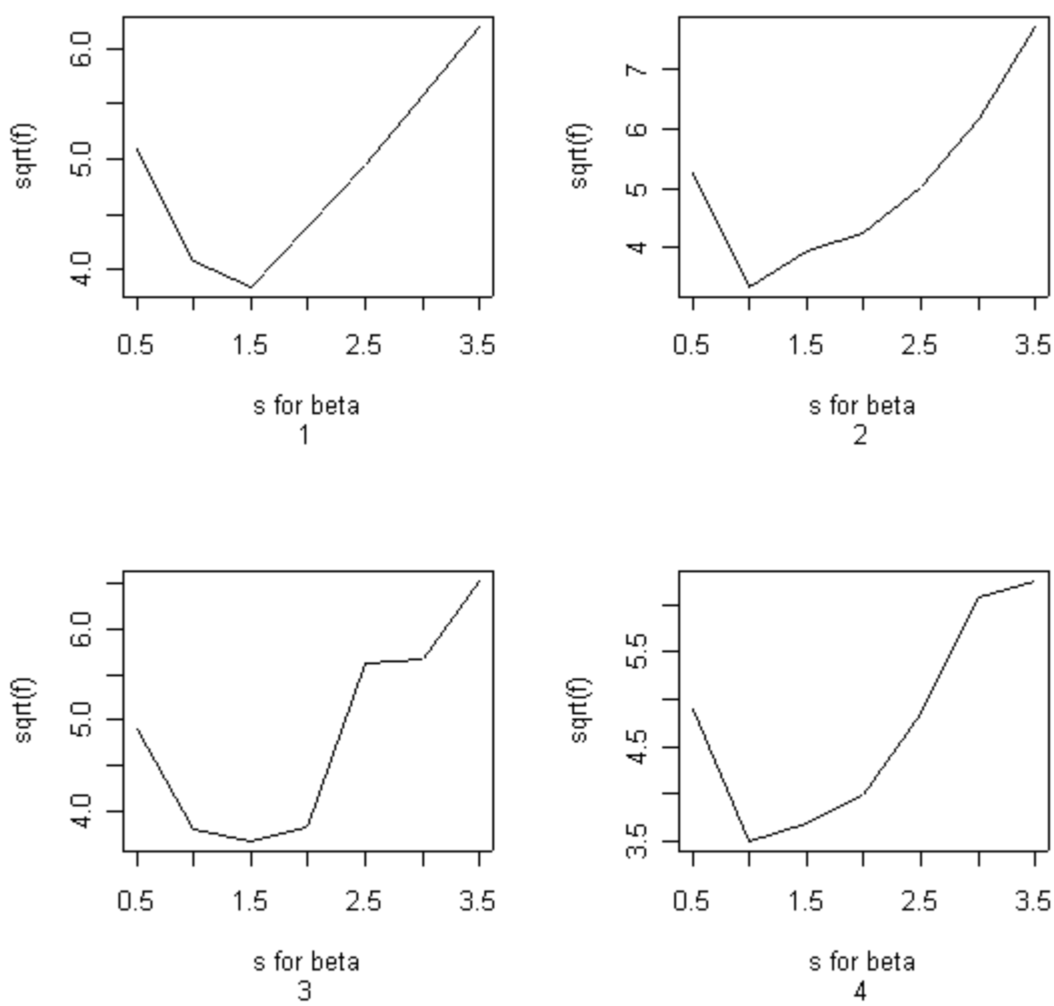
($\hat{\delta}$ and $\tilde{\delta}$ are sample equivalents in the inequality in (1))

Thus, suggestion (b) is incorrect.

2. The random walk Metropolis routine for the Multinomial Logit is implemented in the file 'test_rmnl.R" in the hw directory.

The data are simulated from a multinomial logit with 3 alternatives, two independent variables draws from $Unif\left(-1,1\right)$ and $\beta' = \left(-0.5 \quad 0.5 \quad 0.2 \quad 0.3\right)$

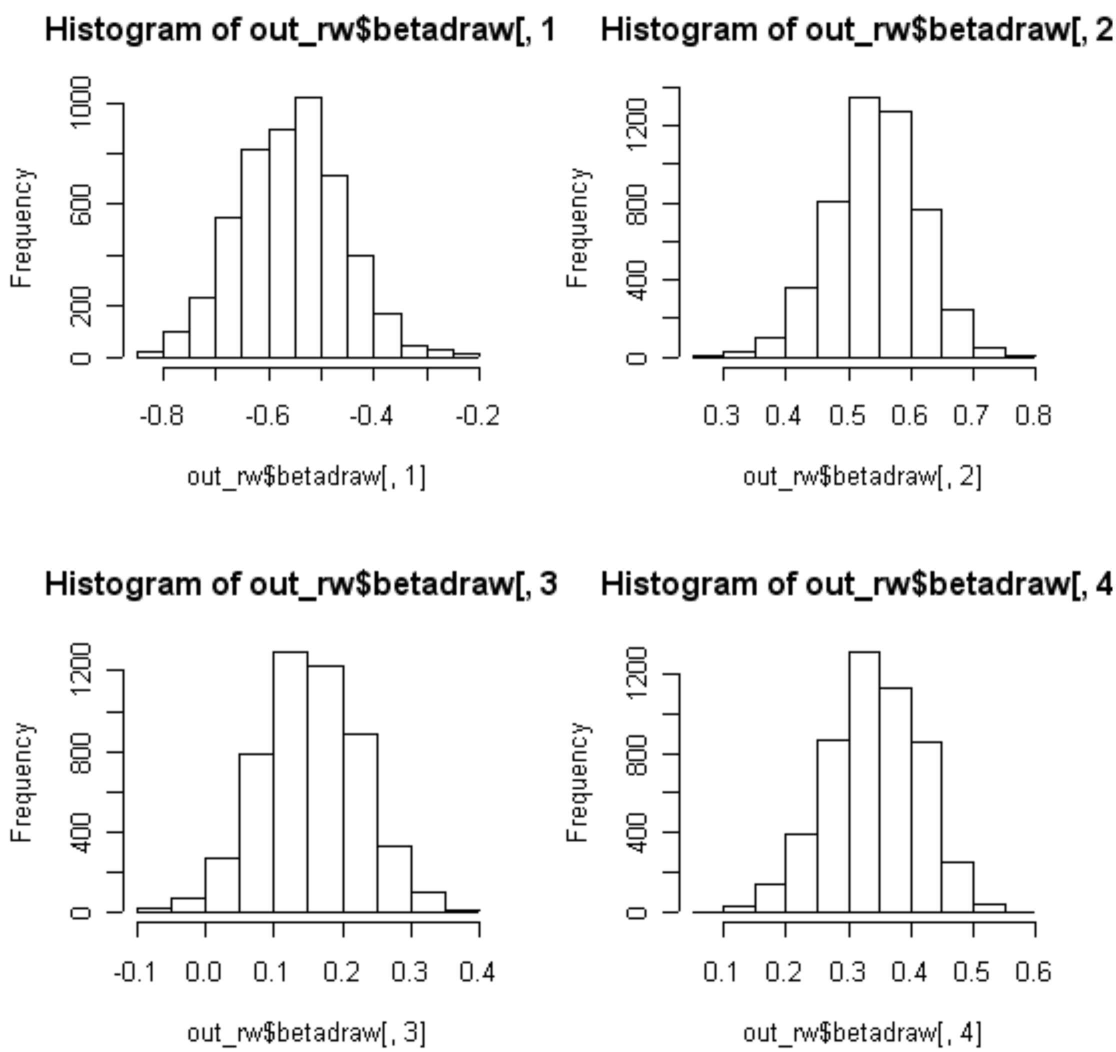
In order to tune the RW algorithm, we plot $\widehat{f_R}$ against s for each of the four elements of β.



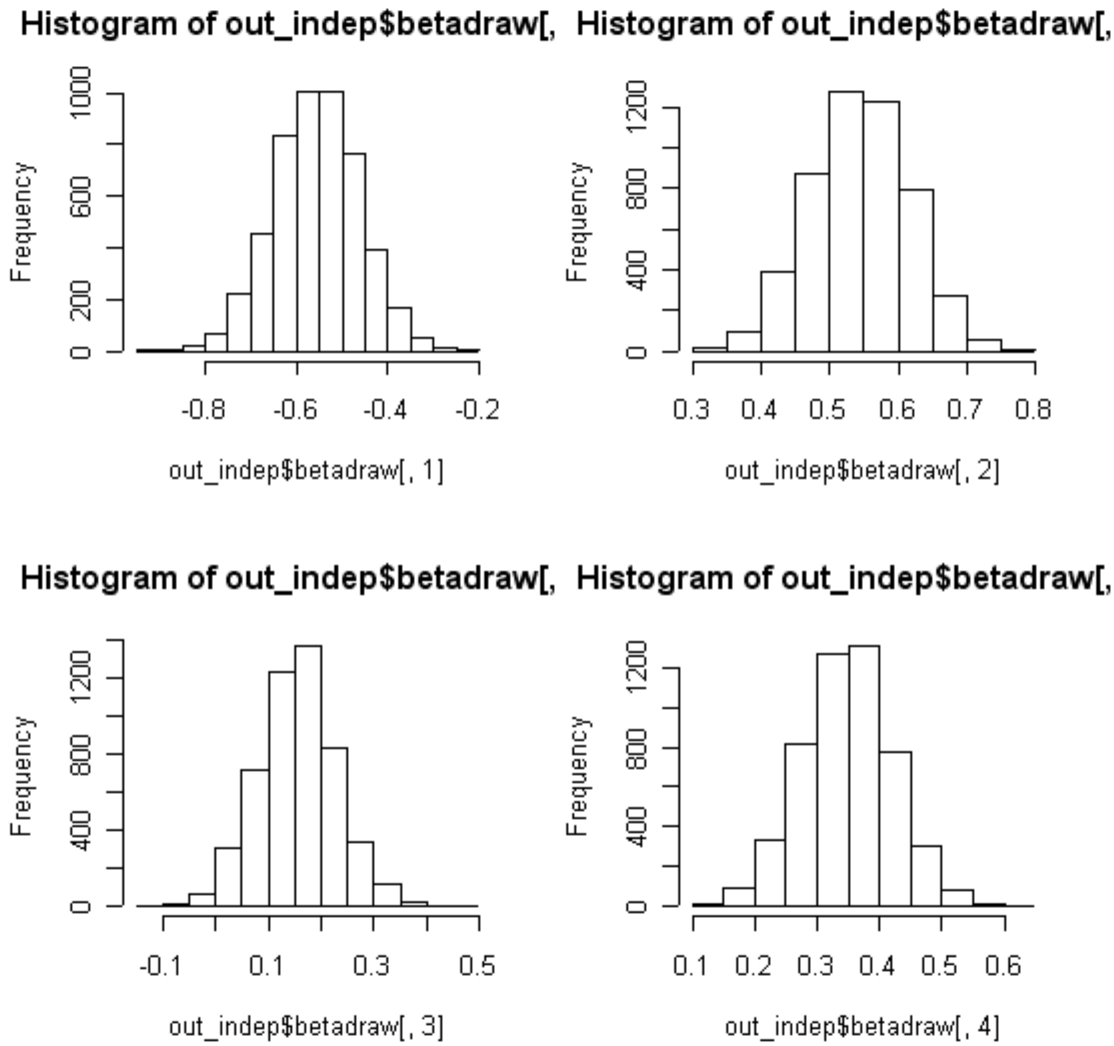
There is no single value of s which is optimal for all the elements of β. In order to assess whether to use $s = 1$ or $s = 1.5$, we could compare the ACF plots for all the elements of β for the two values of s and pick the value of s that seems to give better acfs (if one dominates the other). Or depending on which parameter is of interest, one could pick s such that $\widehat{f_R}$ is minimized for that parameter.

Here, we pick $s = 1.5$. The histograms and ACF plots for the Random Walk and Independence chains (with $\nu = 6$) are given for comparison.

Random Walk Metropolis

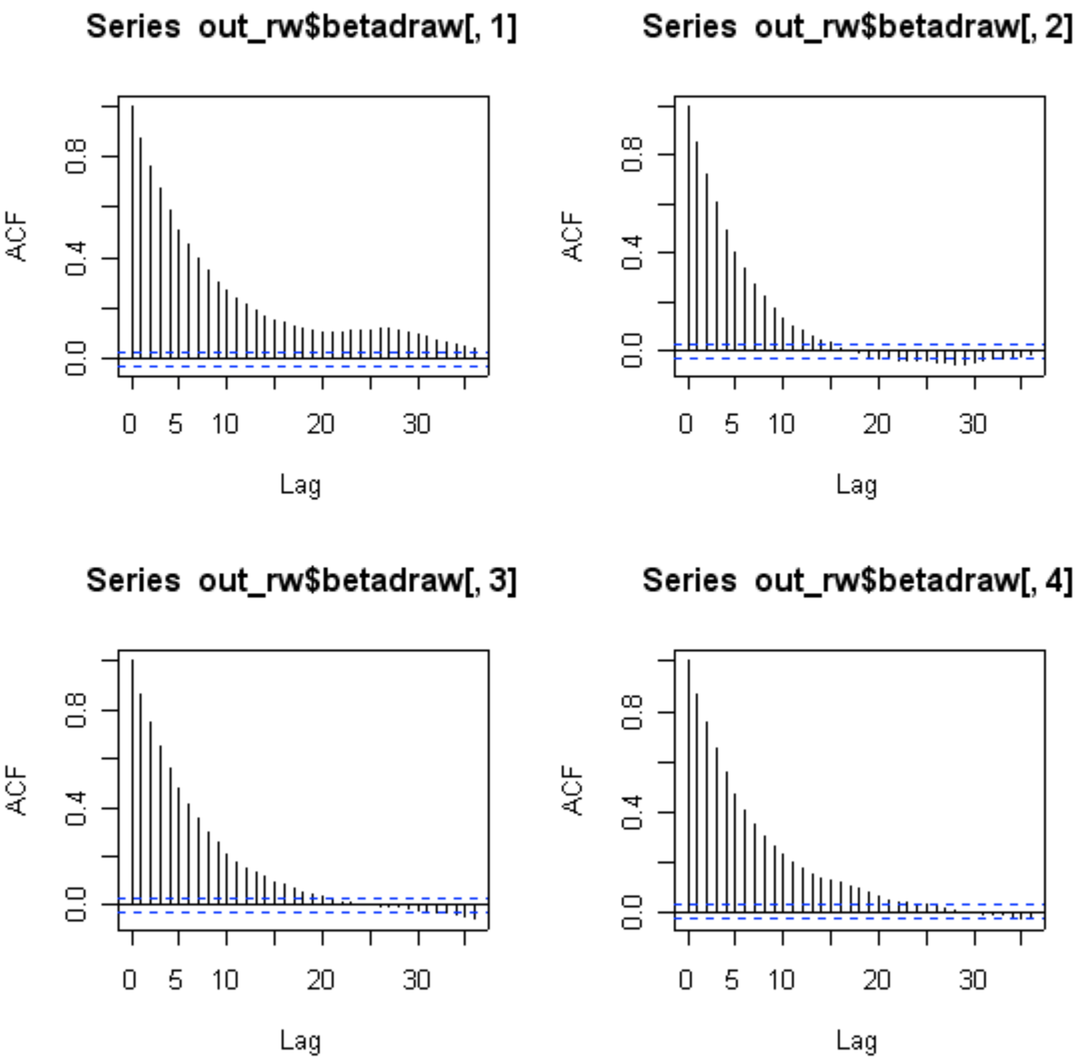


Independence Metropolis



The posterior distributions look very similar for both the Random Walk and the Independence chains. We can also compare the ACF plots for the two chains. Clearly, the Independence chain has much lower autocorrelations than the random walk.

Random Walk Metropolis



Independence Metropolis